**Benefit-Cost Analysis Notes**

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Note to Students: These are intended as a guide to myself—to jog my memory about what I want to discuss in class. They don’t consist of complete thoughts. They are not intended as a study guide. That something is written here, does not mean it will be the correct answer on a test. There may be numerous typos. They are a work in progress. *USE WITH CAUTION!*

**Detailed Topic Outline** (Timing is tentative and will change.)

1. Introduction (Chapters 1-2, Week 1)
   1. Definition of net benefits
   2. Perspectives: Analysts, Guardians, Interest Group Politics
   3. Standing
   4. Conceptual Foundation- Efficiency and highest valued use
      1. Willingness to Pay
      2. Opportunity Cost
      3. Pareto Efficiency
      4. Kaldor–Hicks efficiency
      5. BCA Shortcomings
2. Microeconomic Foundations (Chapter 3, Weeks 2-4)
   1. Demand, Willingness to Pay, Consumer Surplus
   2. Estimates using elasticity of demand, slope (Chapters 4 and 8)
   3. Supply, Producer Surplus, Factor Surplus
   4. Government budget, taxation, excess burden
3. Primary impacts in output markets (Chapter 5, Weeks 4-5)
   1. Willingness to Pay in Efficient Primary Markets
      1. Price unchanged
      2. Price changed
   2. Willingness to Pay in Distorted Markets
      1. Price unchanged
      2. Price changed
4. Primary impacts in input markets (Chapter 6, Week 6)
   1. Opportunity cost in undistorted markets
      1. Price unchanged
      2. Price changed
   2. Opportunity cost in distorted markets
      1. Price unchanged
      2. Price changed
5. Secondary Markets (Chapter 7, Week 7)
   1. Efficient secondary markets
      1. Price unchanged
      2. Price changed
   2. Distorted Secondary Markets
   3. Local Perspective on Secondary Market Effects
   4. Multiplier effects
6. Discounting (Chapter 9, Week 8)
   1. General definition and formula
   2. Annuity formulae
   3. Applications
      1. Time value of money
      2. Inflation
      3. Growth
      4. Depreciation
      5. Obsolescence
   4. Non-identical timeframes
      1. Roll Over Method
      2. Equivalent Annual Net Benefit
   5. Long lived projects and horizon value
   6. Sensitivity analysis

***We will take the midterm at this point.***

***We will devote significant class time to project related activities from here on, with the schedule dictated by the needs of the groups.***

1. Social Discount Rate (Chapter 10, Week 9)
   1. Alternative safe interest rates
   2. Social consumption smoothing
   3. Optimal growth rate approach – veil of ignorance
2. Uncertainty and Information Value (Chapter 11, Weeks 10-12)
   1. Contingencies
   2. Expected value and expected net benefit
   3. Attitudes toward risk and risk pooling
   4. Decision Problems as Normal Form Games Against Nature
      1. Uncertainty with Impacts over Time
      2. Expected net present value
      3. Decisions up front
      4. Decisions at different points in time
         1. Decision Tree and backward induction
         2. Quasi-Option Value or Real Option Value
   5. Sensitivity Analysis
   6. Value of Information
3. Option Price and Option Value (Chapter 12, Week 13)
4. Existence Value (Chapter 13, Week 14)
5. Project Presentations (Week 15)

**1. Introduction (Chapters 1-2, Week 1)**

How to evaluate of a project is worth pursuing, or, which to pursue from among many options?

Private Company: Profit = Revenue – Cost

What about government projects? Non-Profits?

Net Social Benefits = Social Benefits – Social Costs

NB=B-C

Cost: All opportunity costs

Discuss difference between social and private benefits and costs (externalities, opportunity costs)

Typical Steps

What are the alternatives?

Identifying all important impacts and all groups notably impacted is tough

From whose perspective? Who has standing?

Predict, quantify, monetize, discount impacts

Sensitivity Analysis

Recommendation

Types: *Ex ante*, *ex post*, both

BCA or CBA?

Bureaucratic and Political Perspectives

*Analysts*: (us)

*Guardians*: e.g., controllers. See revenues as benefits, expenditures as costs, skeptical of CBA as a justification for spending. Tend to ignore non-financial benefits and costs.

*Spenders*: tend to regard all expenditures on constituents as benefits, not costs. View outlays by constituents as costs.

*Interest Group Politics*: politicians prefer concentrated benefits, diffuse cost

Demand for BCA:

Private Firms:

Non Profits:

Government: e.g. required for a regulatory changes, to evaluate some legislation, obviously choosing among projects

Courts: assess damages, decide anti-trust cases

Conceptual Foundation

Framework to measure efficiency: resources employed in their highest valued uses.

Pareto Efficiency

We can all agree on Pareto efficiency

Most potential policy changes / projects are not Pareto efficient. Does that mean they should not be pursued?

Kaldor–Hicks efficiency. Like *Potential Pareto-Efficiency*.

Money allocation example, graph with status quo

In BCA

Benefits measured using ***Willingness to Pay***

Costs measured by ***Opportunity Cost***

If the sum of all impacted parties ***WtP*** exceeds the sum of the ***OC*** of all inputs, a change is K-H Efficient

Fallacies of Benefit-Cost Ratios

(make up example)

Choosing from exclusive options the one with the highest NB ***IS NOT*** the same as choosing the one with the highest benefit:cost ratio

B:C ratio ***can be manipulated***, e.g. by where negative ***WtP*** is counted

Potential Shortcomings of BCA

Book mentions Arrow’s Impossibility Theorem, but…

1. Dependence on Wealth Distribution. Trouble with K-H efficiency is, compensation is not actually paid.

Marginal utility of a dollar varies with wealth

Present impacts by wealth group? Distributionally weighted? Whose weights?

2. Who has standing?

i) Jurisdictional definitions. What about spillovers?

ii) Exclusion of socially unacceptable preferences

iii) Preferences of future generation? Reflected in current peoples preferences?

3. Technical limits

4. Goals other than efficiency (e.g. distribution)

**2. Microeconomic Foundations (Chapter 3-4,8, Weeks 2-4)**

Demand and Willingness to Pay

Demand, Inverse Demand, Marginal WtP, WtP, and Consumer Surplus

*Practice: Make up a problem*

Compensating variation: Amount individual thinks the change is worth IF they are held constant at their *ORIGINAL* level of utility.

Equivalent variation: Amount individual thinkss the change is worth IF they are held constant at their *NEW* level of utility.

What is the difference? After a price increase, you are poorer, your utility is different.

For a price increase: CV>ΔCS>EV

For a price decrease, EV>ΔCS>CV

If you are not much poorer, the difference is small: modest price change, and small portion of income spent on the good in question.

Elasticity of Demand: 

Using elasticity to estimate ΔCS

Draw it.



*Practice: Make up a problem. Also, use elasticity to get a linear approximation AND a log-linear approximation of demand.*

Supply, Producer Surplus, Factor Surplus

Short run:

From marginal cost to supply.

Area under supply curve is TVC

Revenue less TVC is producer surplus.

This definition of producer surplus is not entirely satisfactory. What about fixed costs? LR with no fixed costs?

Producer Surplus = Profits + Ricardian Rents

What are Ricardian Rents?

Social Surplus and Allocative Efficiency under Perfect Competition

Discussion of Supply and Perfect Competition

Government Budget

Projects also impact government budgets

Expenditures are costs to taxpayers.

So, include net government budget impact, government surplus, GS

Social Surplus and Net Social Surplus

SS=CS+PS+GS

ΔSS=ΔCS+ΔPS+ΔGS

Problem: raising taxes distorts markets (loss of efficiency) AND has administrative costs. Need to include that in ΔSS

Taxation

1) First, show it for general supply and demand curves.

2) Assume horizontal supply curve at P0.

Per unit tax of t. Draw it. 

DWL here is the Excess Burden of Taxation

“Leakage”=EB per Dollar of Tax Revenue

Tax Revenue = R





.

If %ΔQ is small: 

Recall from above: ΔSS=ΔCS+ΔPS+ΔGS

What about the ***marginal*** excess tax burden: ΔSS=ΔCS+ΔPS+(1+METB)ΔGS

Alternative weights?

Useful to show impacts on each group separately—clients/users may weight different groups differently.

Case for intervention depends on either Market Failure or Government Failure

Important: Leakages can occur on the expenditure side too.

*Practice*

Assumptions:

1) ηd=-2

2) ηs=1

3) P0=5 (before tax)

4) Q0=1000 (before tax)

5) t=2.

Notes:

1) with the tax Pd=Ps+t

2) relative to the starting point %ΔPd=(Ps+t)/P0

3) %ΔQd=%ΔQs

4) From these you can solve for Ps, Pd, and Q after the tax.

Problem

1) Calculate tax revenue, the consumer burden, the producer burden, and the excess burden of the tax, and the average rate of leakage.

2) Assume t increases to 2.1. Redo the calculations from (1).

3) Calculate the ratio of the increase in the DWL to the increase in revenue, that is, the incremental rate of leakage when t increases from 2 to 2.1, not the average rate. Explain why this incremental rate differs from the rate in (1).

**3. Benefits and Costs in Primary Output Markets (Chapter 5, Weeks 4-5)**

Primary v Secondary 🡪 Directly Impacted v Indirectly Impacted.

e.g. fleet mpg regulations directly impact auto manufacturers, less directly impact fuel markets, tire markets, bicycle markets, etc…

**Willingness to Pay in Efficient Output Markets**

1) Adding a quantity, q’, small enough that the price is not appreciably changed. Sketch it… (Obviously not literally true)

a) Sold by the government: ΔGS=pq’

b) Given away by the government, resale easy: grantees gain ΔCS=pq’

c) Given away by the government, resale difficult: VERY HARD! Value per unit varies between 0 and p. But, what is the distribution?

Those that get it may not value it much

Could burn money time and effort trying to resell, e.g. standing in line

Half p?

2) Adding quantity, q’, that changes price appreciably.

a) Sold by the government:

D

p

S

Q

Q1

p1

S+q’

Q0

p0

Q1-q’

Price falls to p1.

CS up (p0-p1)Q0 + (Q1-Q0)(p0-p1)/2.

GS up pq’.

PS down (p0-p1)(Q1–q’) + (Q0-Q1+q’)(p0-p1)/2.

b) Given away by the government, resale easy: like case a PLUS grantees gain additional ΔCS=pq’

c) Given away by the government, resale difficult: VERY HARD! Value per unit varies between 0 and p. But, what is the distribution? How is the market price impacted to find impact on firms?

3) Program shifts supply right or down, e.g. port expansion, channel deepening, but does not impact price. Sketch it. All gain is PS. (obviously not literally true)

4) Program shifts supply right or down, enough to impact price.

D

p

S0

Q

Q1

p1

S1

Q0

p0

Qs1

c0

c1

Price falls to p1.

CS up (p0-p1)Q0 + (Q1-Q0)(p0-p1)/2.

PS up (p1-c1)Q1/2 – (p0-c0)Q0/2

**Willingness to Pay in Distorted Markets**

Monopoly – break it up?

Natural Monopoly – regulate price, options?

Ignore book treatment of information asymmetry

Negative Externality – Corrective Tax

Impact on CS, PS, GS, Third Parties

Positive Externality – Corrective Subsidy

Public Goods (like an extreme positive externality)

Addiction

**4. Impacts in Input Markets - Opportunity Costs (Chapter 6, Week 6)**

Labor, land, materials, etc…

What was the value of what would have been done with these resources?

Simplest to use their budgetary cost *moving forward*.

That is not always right, though…

What about sunk costs? E.g., construction already in place?

1) Undistorted input markets and the project has negligible impact on input prices

Value inputs at their market prices (should correspond to budgetary outlays)

2) Undistorted input markets and the project increases input prices

DRAW IT

Has to be a BIG project, or, to use inputs in very restricted supply

(land with good access to a particular intersection, for example)

Note value of (Q0-(Q1-q’)) units just gone. That is OC of those units.

OC of the rest is area under supply.

So, some PS and CS is permanently eliminated

The first part of what book labels G and their B and E represent value no longer captured by those that would have purchased the input.

Budgetary outlay just p1q’

Apply METB to whole expenditure.

Part of expenditure is a transfer to sellers.

3) Inputs purchased in distorted markets: COMPLICATED!

Carefully consider changes in surplus given the details of each case.

Examples:

a) Legal restrictions keeping price below opportunity cost ,

Example: juror compensation. Find the pretax wage rate. Apply METB only to expenditures.

b) Hires in the presence of high labor unemployment.

DRAW IT! Let wr be the reservation wage (vertical intercept)

Market rate times quantity hired is the budget outlay.

Market rate overestimates opportunity cost!

Remaining unemployed has huge costs, some would work much cheaper

No reason to think lowest opportunity cost workers are employed.

Reasonable Guestimate: treat OC cost as (pm-pr)/2

Problem, what is reservation wage?

Assume it is 0. Why is this perhaps reasonable?

c) Purchases from a monopoly

D1=D+q’

p

MR1

Q

Q1

p1

MC

Q0

p0

Q1-q’

D

MR0

p>MC, so budget outlay overstates opportunity cost.

Hard to estimate.

Simplification: assume constant MC, can use elasticity of demand and price to guess MC, if MC data not available.

Increased PS: (p1-MC)Q1-(p0-MC)Q0

Direct OC of resource: q’MC

Additional lost use value: p0(Q0-(Q1-q’))+(p1-p0)(Q0-(Q1-q’))/2

Increased monopoly profit: (p1-MC)Q1-(p0-MC)Q0

Loss of GS: (1+METB)p1q’

Note there is a sizeable transfer from taxpayers to monopolist, but original use value of Q0-(Q1-q’) units is simply gone.

Mention some others: externalities, subsidized markets, etc…

**5. Valuation of Secondary Market Impacts (Chapter 7, Week 7)**

Build a youth sports complex.

Direct: lowers the price of playing (soccer, baseball, whatever)

Indirect: more demand for sports gear

\*\*Often local secondary effects get a lot of political play

**Efficient Secondary Markets with Negligible Price Change**

Can ignore changes in secondary effects if the secondary market is not distorted and price changes in the secondary market are negligible.

e.g. build more fields for youth baseball

Draw the pictures!!! (horizontal supply)

Lower MC in primary market. Lower price in primary market. Higher demand in complement market.

**Note**: demand curves hold prices of substitutes and complements constant.

Counting CS change in both markets is double counting. WHY????

Demand in primary market is based on having the equipment newly purchased in the secondary market. The most consumers would pay for the extra baseball games assumes they get use from the newly purchased equipment, includes the gain they get from using that equipment.

***IF*** you can’t directly measure social surplus in the primary market, you could try to use observed changes in the secondary market(s) to infer changes in the primary market. Plenty of guesswork involved.

**Efficient Secondary Markets with Price Change**

The project or program would have to be HUGE for this to be a concern, or, to impact very small fragile purely local secondary markets.

This one gets subtle

Suppose the secondary market is a substitute, music lessons, with a local upward sloping supply curve

Ds0

p

Ds1

Q

Qs1

ps1

S0

Qs0

ps0

S1

Youth Sports

D\*

c

a

b

Qs~

Dm0

p

Dm1

Q

Q1

pm1

S

Q0

pm0

Music Lessons

Q~

g

f

e

The decline in the effective price of youth sports lowers the demand for music lessons and their price.

This LOWERS the demand for youth sports. \*\*\*Quantity of youth sports must still increase, or demand for music lessons would not be down!

Fall in demand for music lessons is accounted for in original demand for youth sports. Those getting music lessons obviously not harmed by lower price of sports. If price stayed the same, all is good.

BUT, price falls from pm0 to pm1. Loss of PS from price fall exceeds CS gain by (pm0-pm1)(Qm0-Qm~)/2. What to do with this?

Fall in lesson price reduced demand for sports. Which demand to use to value?

If use D0, must subtract area of efg

ΔSS ≈ (ps0-ps1)Qs0 + (ps0-ps1)(Qs~-Qs0)/2 – efg, OR

D\* is called an equilibrium demand schedule.

Often what we have in practice.

If use it, already approximately includes loss in secondary market:

ΔSS ≈ (ps0-ps1)Qs0 + (ps0-ps1)(Qs1-Qs0)/2,

Depends only on observed quantities and prices in primary markets

abc ≈ egf

Did your demand estimate for sports hold music lesson prices constant or not? If it did, have to allow that adjustment. If it did not, already accounted for it.

\*\*\*If secondary markets are not distorted, and if you directly measure “equilibrium” demand in primary market, ignore all secondary market effects.

**Valuing Benefits and Costs in Distorted Secondary Markets**

Example 1: suppose a regulation raises the costs of driving, but there are negative externalities to gas consumption of x per gallon. Lowering gas consumption by q’ gallons created xq’ units of benefit.

Example 2: suppose two substitute products, one taxed, one not. Taxing the untaxed one increases demand for the taxed one, raising government revenue.

In general: difficult to measure properly quantitatively. Assume safe to ignore. When they are not, takes a lot of work to account for such effects.

Mention Computable General Equilibrium models.

**Indirect Effects of Infrastructure Projects**

e.g. Better shipping from port lowers manufacturers cost which lowers their prices which lower costs for those who use their products as inputs.

Reasonably approximated by primary market analysis unless the indirectly affected markets are severely distorted. Similar to argument for secondary markets—would be double counting, unless seriously changes indirect market prices.

**Local Perspective on Secondary Market Effects**

e.g. demand for local hotels, restaurants, etc… with *multiplier effects*

Absent distortions

For society as a whole, this is wrong. Look at total change in Social Surplus.

If we restrict standing to local residents, benefits to non-locals also can’t be counted as benefits—e.g. CS value of vacations to tourists

If prices don’t change, nothing going on anyway.

If prices increase, some producers better off, but local residents paying higher prices, are worse off.

Remember, adding net change in PS and CS, secondary effects themselves are always a net negative! (Show it.) Why should most of the negative fall on outsiders and the positive on insiders?

Local multipliers dissipate quickly—businesses owned by outsiders, imports, etc…

With distortions

With high and localized unemployment, local effects may matter

Otherwise, in general, secondary effects can be both positive and negative. Need a very detailed analysis to sort them out.

**7. Discounting and Net Present Value (Chapter 9, Week 8)**

Projects have different expenditures and different benefits at different times.

How to compare?

Over the same timeframe

For now, consider the same overall time frame (e.g. all projects have costs and benefits over the same 20 years)

Put them all in net present value terms.

Demonstrate with examples

Define: i: discount rate, t: number of periods (years) from start (0), T: end time

Generally:

 or 

Timing of payments?

Annuity formulae

Assume you have a discount factor δ to be applied each period. The simplest case is a discount rate of i (e.g. the risk free interest rate) and δ=1/(1+i). But, it can be more complex:

Starting with a payment of A at t=1 continuing for T payments with the last at time t=T, the sum of the discounted values, S, is:

S(A,T,δ)=Aδ+Aδ2+∙∙∙+AδT

Multiplying by δ gives

δS(A,T,δ)=Aδ1+Aδ2+∙∙∙+ AδT+AδT+1

The two are the same but for the first term of the first and the last term of the second. Subtracting the second from the first gives

(1-δ)S(A,T,δ)=A(δ-δT+1)=Aδ(1-δT)

Solving gives:

S(A,T,δ)= A(1-δT)[δ/(1-δ)]

A generalized annuity factor, as a function of T and δ, is

a(T,δ)=(1-δT)[δ/(1-δ)]

This discount factor can represent, for example:

Time value of money in nominal terms at interest rate i, δ=1+i

Time value of money in real terms at real discount rate is r, δ=1+g

Inflation at rate m, δ=1+m

Growth at rate g: δ=1+g [Note for this one you have to start with the value at period 0, A0, which grows to the value at A1 in period 1, so A0=A1/(1+g).]

Depreciation at rate d, δ=1-d

Obsolescence at probability after each period, δ=1-f

Combinations, e.g. discounting, obsolescence, growth: δ=(1-f)(1+g)/(1+r)

Four useful things to keep in mind

1) if payments start at 0, not 1, S=A+Aa

2) when T→∞,a→ δ/(1-δ)

3) If δ=1/(1+q) and T→∞ a→δ/(1-δ)=1/y

4) If payments start at 0, not 1, δ=1/(1+y), and T→∞,S→A+A/y=A(1+1/y)

Different Time Frames

Mention Roll Over Method

Equivalent Annual Net Benefit (EANB)

For project p, let Tp be the final year benefits or costs accrue. Then:



Choose the project with the higher EANB

Example

i=0.05

|  |  |  |
| --- | --- | --- |
|  | X | Y |
| t | NB | NB |
| 0 | -10 | -20 |
| 1 | 8 | 25 |
| 2 | 10 | NA |
| NPV |  |  |

Problem: annuity formula start at t=1. What to do?

1. Convert the annuity factor to one starting at t=0



EANBX=

EANBY=

1. Treat them as 3 and 2 year projects respectively, not 2 and 1



EANBX=

EANBY=

Introduce the idea of option value

Inflation

If the inflation rate π, takes $1(1+π) tomorrow to buy what $1 buys today, or $1 tomorrow only worth $1/(1+π)

If inflation rate constant, or have t “average” rate from base t=0: 

Generally, for base t=0: 

Deflators (CPI or PCE deflator)

*Real interest rate*: If invest $1 at rate i for 1 year, and inflation rate is π, after one year you have $(1+i) but can only buy $(1+i)/(1+π), so:

 or 

For small m, or continuous compounding, r≈i-m

***IMPORTANT***: If present costs and benefits in real dollars, must use, r.

Should you use real or nominal (current) dollars?

If you use nominal values, use the nominal interest rate:



Why not just use nominal values? How do you know future nominal values?

What if you just have an estimate of base period prices?

This is the real case. Just use real rate.

Estimating Future Inflation Rates-if you Really Must

Best – Compare nominal rate of interest on government bonds to real rate on inflation indexed bonds of the same term. e.g. 6 versus 2.

(*Begs the question*: why not just use the real rate and base period prices?)

***Note***: Relative price changes are not the same thing as inflation, ***must*** figure directly into the BCA estimates of benefits and costs.

**Very Long Lived Projects**

e.g. In England, some currently used roads laid by the Roman Empire nearly 2 millennia ago

If believe benefits continue indefinitely, use infinite horizon.

What if much more confident about benefits up to T years out (e.g. 20 years):

1) Do NPV through year T, discounting period

2) Do PV of Horizon Value at year T, HT

3) Do sensitivity analysis wrt horizon value

T often chosen to be time before major unpredictable repairs start to arise

Estimating HT

1) Simple projection: basically infinite horizon, but break into 2 parts for sensitivity analysis and to emphasize uncertainty

2) Scrap or liquidation value: if feasible to determine it

3) Depreciated (real or economic) value: just a way to estimate liquidation value

4) Fraction of initial construction cost: if fraction based on economic depreciation, same as 3, if not, senseless.

5) Use 0

Time Declining Discount Rates

Sensitivity Analysis: i, HT, Breakeven Rate (IRR), Break Even HT

**7. Social Discount Rate (Chapter 10, Week 9)**

In perfect markets, everyone adjusts saving and lending until everyone’s marginal rate of time preference equals the marginal rate of return (rate of return on a small marginal investment), and the real market interest rate.

Transactions costs, liquidity constraints, etc… mean markets are not perfect. So, which (real) interest rate to choose is a hard question.

Some options for choosing based on various observed market interest rates

1) Real before tax return on (AAA) corporate bonds. Around 4.5%

An upper limit: tax crowds out consumption, not just investment, and there is some degree of risk premium in corporate bonds

2) Real after tax return on savings. e.g. 10 year t-bill inflation protected, adjusted down for tax rate. Around 1.5% ish. A lower bound. Liquidity constraints, not all financed from consumption, irrationality, etc…

3) Government long term borrowing rate 2.5% or so

4) Some average of 1-3. Weights? Around 2.5-3%

Also: given economic growth, those in the future will be richer. Social consumption smoothing means discount rate should be a bit higher than the pure rate of time preference. Round it up some.

Also: current individuals preferences may not reflect future preferences. What to do with this?

**Optimal Growth Rate Approach**

r=d+ge

d: pure rate of (social) time preference, around 1% (this one is tricky!)

g: growth rate (2%, give or take)

e: social marginal utility of consumption with respect to per capita consumption, around 1.5

So, r=.01+1.5\*.02=3.5%

All are guestimates. Do sensitivity analyses.

**8. Uncertainty and Information Value (Chapter 11, Weeks 10-11)**

Definition of Expected Value

Contingencies: exhaustive, mutually exclusive VS scenarios

Illustrate with rainfall example from book

Need a pdf (discrete v continuous, objective v subjective, frequentist v Bayesian)

Define Expected value: Sum over j of fjxj

Expected Net Benefit: Sum over j of fj(Bj-Cj)

Work an example

We are treating decision maker as risk neutral.

Far simpler than the alternatives, leaves things out, though

Discuss certainty equivalents, risk aversion, risk pooling, insurance

Modeling Risk Management Problems as Games Against Nature

Normal form: table listing options, contingencies, and payoffs

e.g. New Youth Sports Facility

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Area Growth | | | Expected Value |
|  |  | 0.2 | 0.4 | 0.4 |
|  |  | Rapid | Medium | Slow |
| Actions | | Net Benefits (2000 Dollars) | | |
| Begin Construction | Now | 9 | 4 | -2 | 2.6 |
| In 4 Years | 4 | 6 | 3 | 4.4 |
| In 8 years | -2 | 2 | 6 | 2.8 |

Importance of sensitivity analysis

What about a (more) continuous approximation?

***Value of Information -1***

Definition: expected value of the amount your expected payoff would increase if you had that information before making the decision.

In example above, ENPV of best option is 4.4. What would you choose in each state of the world IF you knew it was that state?

Value of Info: 0.2\*9+0.4\*6+0.4\*6 – 4.4 = 6.6 – 4.4 = 2.2

It would be worth 50% of the ENPV to acquire the information with certainty BEFORE the decision.

Usually, we don’t get perfect information. Just refine the degree of uncertainty!

1) Still, points to which parameters more important to study more.

2) Can create a guestimate anyway. How?

Need a guesstimated probability distribution over the probability distributions you will have after the study!

**Uncertainty with Impacts over Time**

If all the decisions are upfront, easy to combine discounting and expected value into expected net present value: 

What if decisions must be made at different points in time? What if benefits and costs depend on what contingencies occurred in the past? What if you get more information over time?

Use Decision Analysis or Decision Theory

Model as an extensive form game against nature

Example: Youth Sports Facility

Build 2 fields now, but unsure if demand will exceed capacity of one, OR

Build 1 now and decide on the other after one year

C1: Cost of building one field

C2: Cost of building two fields at once, C2<<<2C1

: field capacity

n: number of users, n1, n2 (could have more), , 

*v*: benefit per user up to capacity (oversimplified, but a useful approximation)

*f*: probability of n1

Build 2 Now:

ENPV=-C2+(v/i)(fn1+(1-f)n2)

Build 2

-C2

f

vn1/i

1-f

vn2/i

Build 1 Now (and only 1 for the foreseeable future):

ENPV=-C1+(v/i)(fn1+(1-f))

f

Build 1

-C1

vn1/i

1-f

v/i

Decision depends on:

ENPV2-ENPR1=(v/i)f(n2-)-(C2-C1)

Building just one saves cost now, with probability f sacrifices future benefits

Build 1 Now and Maybe 1 Later???

Leaves you the option to learn something first. Problem, C2<2C1. Is it worth it?

***Quasi-Option Value or Real Option Value***

Discuss options, puts, calls.

Should reflect this in the decision problem!

*Build Decision Tree then use backward induction*

f

Build 1

-C1

Build 2

-C2

vn1/i

1-f

v/(1+i)

Build 2nd

-C1/(1+i)

vn2/(i(1+i))

v/(i(1+i))

Stay at 1

f

vn1/i

1-f

vn2/i

Start at the end.

Would build a second, having found high demand, IF:

vn2/i – C1>v/i, (n2-)v/i>C1

If not true, decision is as above. If true, prune accordingly and proceed.

*Payoff if build 1:*

fvn1/i – C1 + (1-f)[v/(1+i) – C1/(1+i) +vn2/(i(1+i))],

fvn1/i – C1 + [(1-f)/(1+i)][v – C1 +vn2/i]

*Payoff if build 2:*

fvn1/i + (1-f)vn2/i – C2

*Build 2, not 1, IF*

fvn1/i + (1-f)vn2/i – C2 > fvn1/i – C1 + [(1-f)/(1+i)][v – C1 +vn2/i]

(1-f)vn2/i – C2 > – C1 + [(1-f)/(1+i)][v – C1 +vn2/i]

[1 – 1/(1+i)](1-f)vn2/i – [(1-f)/(1+i)]v – C2 > –[1+(1-f)/(1+i)]C1

[i/(1+i)](1-f)vn2/i – [(1-f)/(1+i)]v – C2 > –[(1+i+1-f)/(1+i)]C1

(1-f)vn2/(1+i) – (1-f)v/(1+i) – C2 > –[(1+i+1-f)/(1+i)]C1

(1-f)v(n2-)/(1+i) – C2 > –[(1+i+1-f)/(1+i)]C1

(1-f)v(n2-) > (1+i)C2 –(2+i-f)C1

(1-f)v(n2-) + (2C1 – C2) – i(C2 – C1) + fC1 > 0

Can deal with much more complex situations, chain together trees, lots of options, lots of intermediate decisions and dependencies, etc…

Probably much uncertainty about n1, n2, f, v. Plenty of work on sensitivity analysis important.

Some useful figures:

Breakeven: e.g. plot ENPV against f or v

Loci of breakeven pairs of parameter values (ENPV level sets)

e.g. plot v to make ENPV 0 against values of f. Above the line, ENPV>0

Same idea two for equality of any two options

e.g. for all values of f, plot values of C2/C1 to make ENPV equal for building one or two right now.

Sensitivity Analysis:

Important/Illustrative Scenarios (Best Case, Worst Case)

Monte Carlo

**9. Option Price and Option Value (Chapter 12, Week 13)**

Special use of the terms in CBA

Expected surplus, E(S), is not really the correct benefit measure under uncertainty

Complicated by risk aversion

Instead, conceptually, ask everyone impacted what is their WtP for the policy/program under consideration before they know which contingency will occur. Call this the “Option Price”

Option Price may be more or less then E(S). It is the individuals CE over the lottery they get under the policy

Add all the Option Prices together to get the aggregate WtP

Option Value = Option Price - E(S)

Why this name? Because of one particular case among many possible:

Maybe, some chance you might use something (national park) in some contingency, but since you have not used it, it is not captured in directly measured E(S).

But, you are willing to pay something to maintain the option of using it in the contingency you would want to.

Is it another benefit class? Or, just a bias in E(S)?

Practical Terms: we use E(S) because it is usually the easier of the two to estimate. Just know it is (probably slightly) biased (up or down both possible).

**10. Existence Value (Chapter 13, Week 14)**

Active use value vs passive use value (nonuse value)

Active:

Rivalrous consumption

Non Rivalrous Consumption, can be direct or in direct consumption,

Passive

Pure existence value, inherent worth

Altruism (use to current or future generations)

Option Value: may use it in the future

Don’t like the books treatment here. All these things in option value the way the book describes it in Ch8. Quasi Option value is properly a (probability) of active use. Place it as passive just because observing use may not reveal it.

How to estimate it????